Lecture 13. ***Investigation of absolute stability of nonlinear ACS by Pópov's method. Theorem of Pópov***

13.1. Concept of absolute stability (stability "in general") nonlinear ACS

A big possibility to investigate stability and even quality of nonlinear systems were opened by the criterion of absolute stability, offered by Roman scientist *V.M. Pópov (В.М. По́пов)* (1960). Mostly it is because of its geometric interpretation, which allows involving frequency methods of investigation of the class of nonlinear system considered here.

Let's a nonlinear system of the following form is given:

$$\dot{x} = Ax + bu \tag{6.16}$$

$$u = f(y) \tag{6.17}$$

$$y = C^T x \tag{6.18}$$

$$f(0) = 0 \tag{6.19}$$

$$0 \le \frac{f(y)}{y} \le k. \tag{6.20}$$

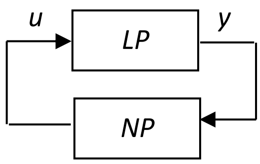

Fig. 6.1. Presentation of a nonlinear system

Equation (6.16)-(6.18) may be written as the following:

$$\begin{cases} \dot{x} = Ax + bf(y) \\ y = C^T x \end{cases}.$$

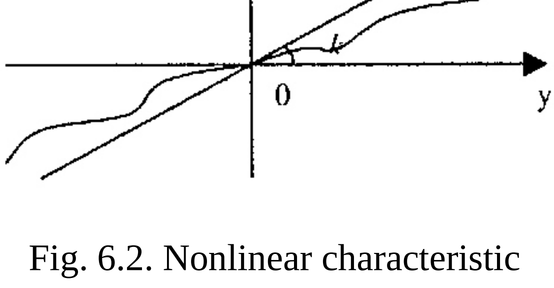

Fig. 6.2. Nonlinear characteristic

Equations (6.19), (6.20) show, that nonlinearity pass through the beginning of coordinates and nonlinear characteristic must be deposed inside the linear angle *(0, k)* (fig. 6.2).

Definition: "Absolute" stability is stability "as a whole" at any nonlinear characteristics of equation (6.17), satisfying to conditions (6.19) and (6.20).

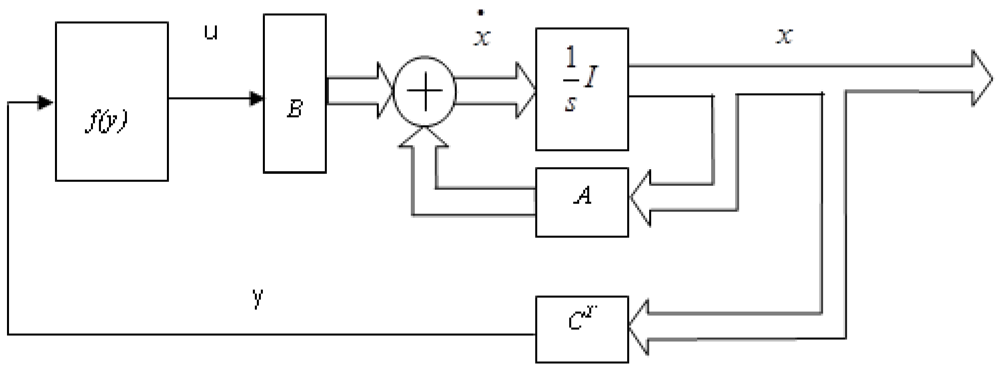

Fig. 6.3. Matrix structural scheme of the system

13.2. Investigation of absolute stability by Pópov's method

Let us pass to Laplace's transformation of equation (6.16):

$$sx = Ax + bu(s)$$
$$(sI - A)x = bu(s).$$
$$x = (sI - A)^{-1} bu$$

We will get matrix transformation function by all variable conditions. For this we will write equation (6.18) in the form of Laplace's transformation:

$$y = C^T (sI - A)^{-1} bu .$$

Scalar transient function by input can be defined as the following:

$$W(s) = \frac{y(s)}{u(s)} = C^T (sI - A)^{-1} b. \qquad (*)$$

Here are $\operatorname{Re} s_i(A) < 0, \forall i = \overline{1, n}.$

Then nonlinear ACS is presented as (fig. 6.4). *LP presents W(s) and NP -- f(y):*

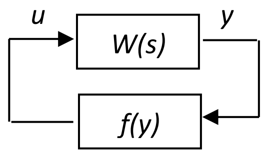

Fig. 6.4. Integrated structure of nonlinear ACS

To this structure Pópov’s technique is applicable through its function:

$$\prod(j\omega) = (1 + jq\omega)W(j\omega) + \frac{1}{k},$$

where $\operatorname{Re} s_i(A) < 0, \forall i = \overline{1,n}$ is a real part of the characteristic roots of matrix *A* polynomial, the value of which is obligatory negative.

For balance of a nonlinear system with a stable linear part, it is sufficient to fulfill requirements, generated in the following theorem.

THEOREM of Pópov

If the real part of the characteristic roots of matrix *A* polynomial is negative, then for absolute stability of the system in angle (0,*k*) it is enough to have such a real final number *“q”,* that at all $\omega > 0$ fulfill the requirement:

$$\operatorname{Re}\prod(j\omega) = \operatorname{Re}\left[(1 + jq\omega)W(j\omega) + \frac{1}{k}\right] > 0, \qquad (6.21)$$

it means that the real part of Popov’s function must be positive.

Note 1. If transient function has one zero pole, then the additional requirement must be satisfied:

$$\lim_{\omega \to 0} \operatorname{Im} W(j\omega) = -\infty.$$

Note 2. If transfer function has two zero poles, then two additional requirements must be satisfied:

a) $\lim_{\omega \to 0} \operatorname{Re} W(j\omega) = -\infty;$

b) $\lim W(j\omega) < 0$ at small ω.

To verify absolute stability by Popov it is necessary to construct modified frequency characteristic of the linear part of the system.

Peculiarities of the modified characteristic are the following: its real part is equal to the real part of the initial characteristic; its imaginary part is equal to the imaginary part of the initial characteristic multiplied by ω.

Frequency characteristic of the initial system is:

$$W(j\omega) = \mathrm{Re}W(j\omega) + \mathrm{Im}W(j\omega).$$

Modified frequency characteristic of the system is equal to:

$$N(j\omega) = \mathrm{Re}\,N(j\omega) + \mathrm{Im}\,N(j\omega),$$

where $\mathrm{Re}\,N(j\omega) = \mathrm{Re}W(j\omega) = X$, $\mathrm{Im}N(j\omega) = \omega \mathrm{Im}W(j\omega) = Y.$

Geometrical interpretation of Pópov's theorem

For absolute stability of system (6.16), (6.17), (6.18) the sufficient condition is that it is possible to draw a straight non horizontal line through the point of the real axis with coordinates $\left(-\frac{1}{k}, j0\right)$ in plain *(X, Y)* in such a way that the modified frequency characteristic *N(jω)* does not cross this straight line (but it can have common points with it).

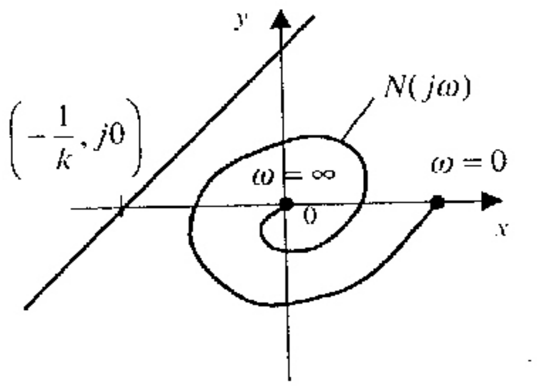

Fig. 6.5. The system is absolute stable

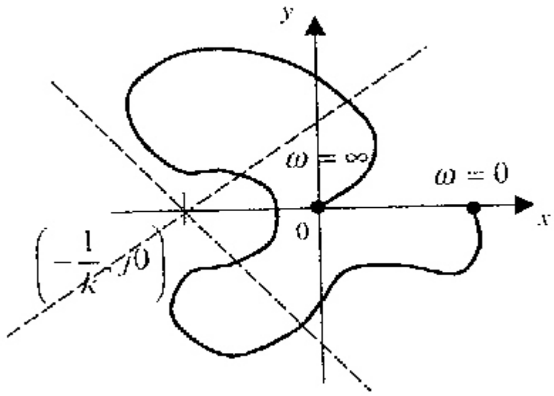

Fig. 6.6. No absolute stability